An Edge-Preserving Super-Precision for Simultaneous Enhancement of Spacial and Grayscale Resolutions

Hiroshi Hasegawa† Toshinori Ohtsuka‡ Isao Yamada‡ Kohichi Sakaniwa‡
† Dept. of Electrical Engineering and Computer Science, Nagoya University
‡ Dept. of Communications & Integrated Systems, Tokyo Institute of Technology
E-mails: hasegawa@nuee.nagoya-u.ac.jp { ohtsuka, isao, sakaniwa } @comm.ss.titech.ac.jp

Abstract—In this paper, we propose a method that recovers a smooth high-resolution image from several blurred and roughly quantized low-resolution images. For compensation of the quantization effect we introduce a measure of smoothness originally used for suppression of block noises in a JPEG compressed image [Schultz & Stevenson ’94]. With a simple operator that approximates to the convex projection onto constraint set defined for each quantized image [Hasegawa et.al. ’05], we propose a method that minimizes these cost functions, which are smooth convex functions, over the intersection of all constraint sets, i.e. the set of all images satisfying all quantization constraints simultaneously, by using hybrid steepest descent method [Yamada & Ogura ’04]. Finally in the numerical example we compare images derived by the proposed method, POCs based conventional method, and generalized proposed method minimizing smoothed total variation and energy of output of Laplacian.

I. INTRODUCTION

A great deal of effort has been devoted to derive a high-resolution image from degraded low-resolution images having potential redundancy [1, 2]. Such a technique is called super-resolution and its application ranges from HDTV to satellite/medical imaging.

Recently several challenges are shown to improve not only spacial resolution but also precision of each pixel, namely bit-depth. In this paper, we call techniques to tackle such a problem super-resolution. Gunturk et al. demonstrated that it is possible to compensate effect of rough quantization by using multiple images [3]. Super-resolution of MPEG video [4–6] is also an example of the super-precision, because it is nothing but compensation of quantization error in the MPEG compression that consists of rough quantization after subtraction of known vector, motion compensation, and orthogonal transform, discrete cosine transform.

Since the quantization constraint leads to numerous linear inequalities and the recovered image is large in general for super-resolution, simple iterative scheme to derive a solution is desirable. Indeed, conventional methods searches for an image satisfying all given quantization conditions [3, 4] by Projection Onto Convex Sets(POCS) [7], that consists of sequential convex projections onto half spaces defined by the inequalities. The other approach [6] modifies the original problem to unconstrained maximization of multidimensional Gaussian function, which approximates quantization error, so that simple scheme such as a steepest descent method resolves the problem. However, the set of all candidates of solution is generally broad therefore quantization interval would be large for higher compression rate. In such a case, one of the most natural approaches would be finding a visually natural image among the set of all images satisfying given quantization constraints. Indeed, ML-POCS [8] provides such a image by iterative convex projections onto a convex set satisfying all of quantization constraints and update to steepest descent directions. However, the proof in [8] only guarantees convergence to the optimal solution only if the convex projection onto a convex set satisfying all of quantization constraints can be computed, although the computation is difficult in general because different blur and shift parameters are assumed.

In this paper, we propose a method that pursues a smooth high-resolution image among the set of all possible candidates defined by differently blurred and roughly quantized low-resolution images. To derive a smooth image while keeping edge information, we introduce a differentiable function that is originally used for suppression of block noises in a JPEG compressed image [9]. By combining hybrid steepest descent method [10] with a simple operator that approximates to the convex projection onto constraint set defined for each quantized image [11], the proposed method minimizes these functions over the feasible set, the set of all images satisfies all quantization constraints. Finally we demonstrate the effectiveness of the proposed method by showing a comparison of images derived from the proposed method, POCS based method, and generalizations of the proposed method to minimize smoothed total variation and energy of output of Laplacian.

II. PRELIMINARIES

A. Notations

Let \( \mathbb{R} \) and \( \mathbb{Z} \) be the set of all real numbers and integers, respectively. Let \( \mathcal{H} \) be a Hilbert space equipped with its inner product \( \langle \cdot, \cdot \rangle_{\mathcal{H}} \) and induced norm \( \|x\|_{\mathcal{H}} := \langle x, x \rangle_{\mathcal{H}}, \forall x \in \mathcal{H} \). A set \( C \subset \mathcal{H} \) is convex provided that \( \forall u, v \in C, \forall \nu \in (0, 1), \nu u + (1 - \nu) v \in C \). For an image \( x \in \mathcal{H} \), the convex projection \( P_C : \mathcal{H} \rightarrow C \) assigns every \( u \in \mathcal{H} \) to the unique point \( P_C(u) \in C \) such that \( d_C(u, C) := \min_{v \in C} \| u - v \|_{\mathcal{H}} \). Given a nonempty closed convex set \( C \subset \mathcal{H} \), the convex projection \( P_C : \mathcal{H} \rightarrow C \) assigns every \( u \in \mathcal{H} \) to the unique point \( P_C(u) \in C \) such that \( d_C(u, C) := \min_{v \in C} \| u - v \|_{\mathcal{H}} \). Let \( g : \mathcal{H} \rightarrow \mathbb{R} \) be a continuous convex function. In this case, for every \( x \in \mathcal{H} \), there exists a vector \( t \in \mathcal{H} \) satisfying \( (y - x, t)_{\mathcal{H}} + g(x) \leq g(y) \). Such a \( t \in \mathcal{H} \) is called a subgradient of \( g \) at \( x \). The set of all subgradient of \( g \) at \( x \) is called subdifferential of \( g \) at \( x \) denoted by \( \partial g(x) \). We often denote a selection of subgradient by \( g'(x) \in \partial g(x) \) because it is a natural generalization of gradient of \( g \) at \( x \).
Suppose that \( g \) has its nonempty level set: \( \text{lev}_{<g} := \{ x \in \mathcal{H} \mid g(x) \leq 0 \} \), which is automatically closed convex set. Then a mapping \( T_{sp(g)} : \mathcal{H} \rightarrow \mathcal{H} \) defined by

\[
T_{sp(g)}(x) := \begin{cases} 
\frac{x - g(x)}{\|x\|^2} & (x \notin \text{lev}_{<g}) \\
x & (x \in \text{lev}_{<g}) 
\end{cases}
\]

is called a subgradient projection (relative to \( g \)). \( T_{sp(g)} \) is a computationally efficient approximation of \( P_{\text{lev}_{<g}} \).

### B. Super-Precision

For all vectors \( u := (u_1, \ldots, u_P) \), \( v := (v_1, \ldots, v_P) \) in a \( P \)-dimensional Euclidean space \( \mathbb{R}^P \), its inner product and induced norm are respectively defined by \( \langle u, v \rangle := \sum_{k=1}^{P} u_k v_k \) and \( \|u\| := \sqrt{\langle u, u \rangle} \). Hereafter we assume that \( u := (u_1, \ldots, u_P) \in \mathbb{R}^P \), its \( m \)th component \( u_m \) is equivalently denoted by \( u(m) \). Let \( M, N \), and \( L \) be positive integers. Suppose that we have a vector \( x \in \mathbb{R}^{LM \times LN} \) and a sequence \( (y_k)_{k \in \mathbb{I}} := (1, \ldots, k) \subset \mathbb{R}^{M \times N} \) derived through lexicographically reordering pixels of an unknown \( LM \times LN \)-high-resolution images and observed \( M \times N \) low-resolution images, respectively. Each low-resolution image is assumed to be generated by:

\[
y_k = DH_k x \quad (k \in \mathbb{I}),
\]

where \( H_k \in \mathbb{R}^{LM \times LN \times L^{2MN}} \) stands for a degradation such as blur and \( D \in \mathbb{R}^{M \times N \times L^{2MN}} \) changes resolution by averaging each \( L \times L \) region. Then each low-resolution image \( y_k \) is quantized after conversion

\[
z_k := Q(T_k(y_k)) \quad (k \in \mathbb{I})
\]

where \( Q \) is a quantization operator with quantization intervals \( \{q_m\}_{m=1}^{MN} \) such that

\[
(Q(u))(m) = q_m \times \left[ \frac{u(m)}{q_m} + \frac{1}{2} \right] \quad (1 \leq m \leq MN),
\]

and \( T_k(y) := S_k(y) + t_k \) is an operator that consists of known linear transformation \( S_k : \mathbb{R}^{MN} \rightarrow \mathbb{R}^{MN} \) and known constant shift \( t_k \in \mathbb{R}^{MN} \). A typical example of such an operator \( T_k \) is the MPEG compression scheme where \( S_k \) is the DCT transform and \( t_k \) is the motion estimation. If a camera system has a large quantization interval because of limited system capability, it would be the other example such that \( T_k = I \). Hereafter we assume that \( T_k = I \) for notational simplicity. This assumption is justified because \( t_k \) is nothing but an offset of the quantization and \( T_k \) can be a part of the linear operator \( DH_k \) in (1).

Throughout this paper, we consider next problem: recover \( x \) from given quantized datas \( (z_k)_{k \in \mathbb{I}} \subset \mathbb{Z} \) and sufficiently estimated \( (H_k)_{k \in \mathbb{I}} \).

The first solution is given by Gunturk et.al. [3]. First they define constraint sets

\[
C_{(k,m,n)} := \{ x \in \mathbb{R}^{LM \times LN} \mid (QDH_k x)(m,n) = z_k(m,n) \}
\]

with \((m,n)\)th pixel \((x(m,n))\) of the image \( x \). Each \( C_{(k,m,n)} \) is a halfspace and convex projection \( P_{(k,m,n)} : \mathbb{R}^{LM \times LN} \rightarrow \mathbb{R}^{LM \times LN} \) onto it can be easily computed. Then they characterize the set of all candidates of solutions, which are images coincide to the observed images \( (z_k)_{k \in \mathbb{I}} \) after degradation by the operator \( QDH_k \), by the intersection

\[
\bigcap_{k \in \mathbb{I}} C_{(k,m,n)} = \left\{ x \in \mathbb{R}^{LM \times LN} \mid \max g_k(x) \leq 0 \right\}.
\]

As we stated in the end of previous section, further selection in the intersection to pursue smoothness is necessary. In this

\[
x_{n+1} = P_{C_{(k,m,n)}} \cdots P_{C_{(1,\ldots,2)}} P_{C_{(1,\ldots,1)}} x_n \quad (n = 0, 1, \ldots)
\]

for any \( x_0 \in \mathbb{R}^{LM \times LN} \).

If large quantization interval \( q_m \) is employed for higher compression rate or limitation of sensor ability, the intersection becomes broad in general and includes visually unnatural images. A candidate to avoid of selection of such images would be introduction of some additional measurement of smoothness. In the next section we present a method that gives a smooth image satisfying all of the given quantization constraints.

### III. PROPOSED EDGE-PRESERVING SUPER-PRECISION SUBJECT TO QUANTIZATION CONSTRAINTS

Let constraint sets in terms of quantized datas \( (z_k)_{k \in \mathbb{I}} \subset \mathbb{Z} \) be

\[
C_k := \{ x \in \mathbb{R}^{LM \times LN} \mid QDH_k x = z_k \}
\]

\[
= \bigcap_{m=1}^{M} \bigcap_{n=1}^{N} C_{(k,m,n)} \neq \emptyset \quad (k \in \mathbb{I}).
\]

An equivalent expression of each set as a level set is given by

\[
\text{lev}_{<g}(g_k) = \left\{ x \in \mathbb{R}^{LM \times LN} \mid \left( QDH_k x - z_k \right)(m) < q_m \right\} \subseteq \mathbb{R}^{LM \times LN}
\]

\[
= C_k.
\]

A subgradient projection \( T_{sp(g_k)} \) relative to each \( g_k \) is given by the following scheme [11]. Let

\[
\Psi_k := \left\{ x \in \mathbb{R}^{LM \times LN} \mid \langle x, (DH_k)^T \Omega_k x \rangle \geq \langle p_{DH_kC_k}(DH_k x), \Omega_k x \rangle \right\},
\]

where \( DH_k C_k := \{ DH_k x \mid x \in C_k \} = \{ u \in \mathbb{R}^{MN} \mid u(m) = z_k(m) + [\text{let } 2 - \frac{1}{2^m}], p_{DH_kC_k} : [\mathbb{R}^{MN} \rightarrow \mathbb{R}^{MN} \text{ is the convex projection onto hyper-cuboid } DH_k C_k, \text{ and } \Omega_k x := P_{DH_kC_k} (DH_k x) - DH_k x. \}

Then an operator approximating the convex projection onto \( C_k \) given by

\[
T_{sp(g_k)}(x) := \Psi_k(x)
\]

\[
:= x + \left\{ \begin{array}{ll} 0 & \text{if } DH_k x \in C_k, \\ \frac{\| \Omega_k x \|^2}{\| (DH_k)^T \Omega_k x \|^2} (DH_k)^T \Omega_k x & \text{otherwise} \end{array} \right\}
\]

The set of all candidates of the solution characterized as an intersection

\[
\bigcap_{k \in \mathbb{I}} C_k = \left\{ x \in \mathbb{R}^{LM \times LN} \mid \max g_k(x) \leq 0 \right\}.
\]
paper we use the following differentiable function for the measurement of smoothness.

**a measurement of smoothness for suppression of JPEG block noise [9]**

The objective function to be minimized is

\[
\log P(x) = \sum_{m=1}^{L M} \sum_{n=1}^{L N} 4 \rho(|\delta_k(x)|(m,n))
\]

where a smooth approximation of \(| \cdot |\)

\[
\rho(t) := \begin{cases} 
\frac{t^2}{2\alpha t} & \text{if } |t| < \alpha \\
-\alpha^2 & \text{otherwise}
\end{cases}
\]

with fixed small \(\alpha > 0\), and

\[
(\delta_1(x))(m,n) := x(m,n+1) - 2x(m,n) + x(m,n-1)
\]

\[
(\delta_2(x))(m,n) := \frac{1}{2}x(m-1,n+1) - x(m,n) + \frac{1}{2}x(m+1,n-1)
\]

\[
(\delta_3(x))(m,n) := x(m+1,n) - 2x(m,n) + x(m-1,n)
\]

\[
(\delta_4(x))(m,n) := \frac{1}{2}x(m+1,n-1) - x(m,n) + \frac{1}{2}x(m-1,n+1).
\]

This measurement of smoothness \(\log P(x)\) in (2) satisfies a condition so called edge-preserving (For definition of edge-preserving, see [5]).

After we propose a method that minimizes the measurement over the intersection. This minimization is resolved by direct application of the following fact.

**Fact 1:** A version of hybrid steepest descent method for quasi-nonexpansive mapping[10, Prop.6] Assume \(\dim(\mathcal{H}) < \infty\). Suppose \(\Phi : \mathcal{H} \rightarrow \mathbb{R}\) is a continuous convex function with \(\text{lev}_{\leq 0}\Phi \neq \emptyset\). Let \(\Phi'\) be a selection of the subdifferential \(\partial \Phi\) and let \(\Phi'\) be bounded on any bounded set. Suppose \(K\) be a bounded closed convex set such that \(\text{lev}_{\leq 0}\Phi \cap K \neq \emptyset\). Assume the Gâteau derivative \(\Theta' : \mathcal{H} \rightarrow \mathbb{R}\) of \(\Theta : \mathcal{H} \rightarrow \mathbb{R}\) is \(\kappa\)-Lipschitzian over \(K\), i.e. \(\exists \kappa > 0\) such that \(\|\Theta'(x) - \Theta'(y)\| < \kappa\|x - y\|\) for all \(x, y \in K\). Then by using any \(u_0 \in \mathcal{H}\) and any \((\lambda_n)_{n \geq 1} \subset [0,\infty)\) satisfying (H1) \(\lim_{n \rightarrow \infty} \lambda_n = 0\) and (H2) \(\sum_{n \geq 1} \lambda_n = \infty\), the sequence generated by \(u_{n+1} := P_K T_{\lambda_n}(u_n) - \lambda_n + \Theta'(P_K T_{\lambda_n}(u_n))\) satisfies \(\lim_{n \rightarrow \infty} d(u_n, \Gamma) = 0\), where \(T_{\alpha} := (1-\alpha)I + \alpha T_{sp}(\Theta)\) and \(\Gamma := \text{argmin}_{\text{lev}_{\leq 0}\Phi \cap K} \Theta(x) \neq \emptyset\).

(Note: The iterative scheme does not require the inversion of \(\Theta'(x)\) at any \(x \in \mathcal{H}\). This is notable advantage of the hybrid steepest descent method because the inversion of \(\Theta'(x)\) is often computationally intensive even in a simplest case where \(\Theta\) is a quadratic function. The necessity of the conditions (H1) and (H2) for \(\lim_{n \rightarrow \infty} d(u_n, \Gamma) = 0\) is discussed in [10, Remark 1]. The speed of convergence of \((d(u_n, \Gamma))_{n=0}^{\infty}\) can be raised by employing reasonable step sizes \((\lambda_n)_{n \geq 1}\) in the initial stage.)

**Algorithm 1:** (Proposed Super-Precision)

Let \(\Theta = \log P(x)\), \(T_{sp}(\Phi) := T_{sp}(g_k)\) with \(k_0 := \text{argmax}_{k \in T} g_k(x)\), and \(K := [0, \mu]^{L^2 MN}\). Apply Fact 1 to derive the unique minimizer \(x = \log P(x)\) over \(K \cap \bigcap_{k \in T} C_k\).

IV. **Numerical Example**

For the image “Lena” (shown in Fig.1(a)) \(x \in \mathbb{R} \cap [0, 255]^{512 \times 512}\) (\(\mu = 255\) means 8bit/pixel), we have low-resolution images \((y_1, y_2) \in \mathbb{R} \cap [0, 255]^{256 \times 256}\) by application of mutually different sub-pixel shift, 2-D Gaussian blur whose variance is

\[
\sigma^2 = \begin{cases} 
3.0 & \text{for } y_1, \ldots, y_4 \\
6.0 & \text{for } y_5, \ldots, y_8 \\
9.0 & \text{for } y_9, \ldots, y_{12}.
\end{cases}
\]

and downsampling after averaging each \(2 \times 2\) region (namely, \(L = 2\)). Then the low-resolution images are equally quantized with quantization interval \(q_m = 32\) (See Fig.1(b)). This interval means bit-depth is 3bit.

Next we recover a high-resolution image by applying some super-precision algorithm. In this example, for the function \(\rho\), we employ \(\alpha = 3\). In addition to the comparison of the proposed method and the conventional POCs based method [3] (derived images are shown in Fig. 1(d) and Fig.1(c), respectively), we introduce the following differentiable functions.

1) **a smooth approximation of total variation**

A smoothed version of total variation is straightforwardly derived by approximating \(| \cdot |\) by \(\rho\)

\[
stv(x) := \sum_{m=1}^{LM} \sum_{n=1}^{LN} \rho^2(x(m+1,n) - x(m,n)) + \rho^2(x(m,n) - x(m,n+1))^{1/2},
\]

where \(x(M+1,n) := x(M,n)\) for \(n = 1, \ldots, LN\) and \(x(m,N+1) := x(m,LN)\) for \(m = 1, \ldots, LM\), respectively.

2) **energy of output of Laplacian**

A discrete version of Laplacian is given by \(\delta_1(\cdot) + \delta_3(\cdot)\). Being a high-pass filter, minimization of squared output \(\|\delta_1(x) + \delta_3(x)\|^2\) will produce smooth image.

The first function \(stv(x)\) satisfies the condition of edge-preserving whereas the second function \(\|\delta_1(x) + \delta_3(x)\|^2\) does not. Both functions are minimized by Algorithm 1 after replacement of \(\log P(x)\) to them.

It can be verified that the proposed method provides smooth image while keeping edge information compared with the conventional POCs based method. From the viewpoint of PSNR, defined by

\[
\text{PSNR} := -10 \log_{10} \frac{\|x_{org} - x\|^2}{255^2 L^2 MN}
\]

with the original image \(x_{org}\), the proposed method also has better performance. The minimization of smoothed total variation is slightly better in terms of PSNR, however it seems that the derived image is over-smoothed. Indeed, face of the woman seems like a plate. The minimization of energy of output of Laplacian does not provide sufficiently smoothed image and its PSNR is slightly worse than the proposed method.

V. **Conclusion**

In this paper, we proposed a method that recovers a smooth high-resolution image from several blurred and roughly quantized low-resolution images. We introduced the cost function in [9] to pursue smoothness while keeping edge information.
Then it has been shown that iterative operations based on Hybrid steepest descent method generate a sequence that converges to an unique minimizer of the cost function over the set of all candidates characterized by observed quantized images. With numerical example, it was verified that the proposed method provides visually natural images compared with a conventional POCS-based method and some generalizations of the proposed method.

Acknowledgment : The authors would like to express deep gratitude to Prof. K.Sato of Nagoya University for his kind support of this work.

This work was partially supported by JSPS Grant-in-Aid for Scientific Research (“KAKENHI”) No. 17700175.

REFERENCES