Cognitive Radio with Relay of a Primary Signal and Piggyback Modulation

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Abstract—This manuscript proposes a cognitive radio system that shares a frequency with an existing primary system. In the proposed method, a secondary transmitter relays the primary signal and piggybacks its own data on it. Symbol error probabilities of the system are derived analytically. The result shows that the proposed system can communicate at a relatively high speed on the same frequency of the primary system without harm to it.

I. INTRODUCTION

This manuscript considers a cognitive radio system where a primary and a secondary user share the same frequency band. In this scenario, the primary system has a priority of the spectrum and ignores the secondary. On the contrary, the secondary system must monitor a primary signal and changes its transmission and reception parameters to avoid interference to the primary system.

A typical spectrum sharing scheme by cognitive radio is “Dynamic Spectrum Access” (DSA) [1], [2]. In this scheme, the secondary system transmits its signal in white space of spectrum where no primary system is using. In other words, DSA requires white space, or absence of the primary system for its transmission. Another cognitive system is underlay of the secondary signal by Ultra Wide Band [3] or transmission power control [4]. In these schemes, the secondary system controls its power spectrum density very low not to affect the primary system. In addition to DSA and underlay schemes, Devroye et al. proposed a concept of the secondary system that cooperates the primary system one-way [5]. They made theoretical discussion on achievable rate by using dirty paper coding (DPC) for pre-coding theoretically.

Following these studies on cognitive radio, this manuscript proposes a practical cognitive radio system which collaborates with the primary system one-way. In the proposed system, the secondary transmitter relays the signal of the primary system and put its own information bearing signal on the relayed primary signal. With this scheme, the data rate of the secondary system has to be low to keep its bit energy high enough with very low power spectrum density. In the proposed scheme, the piggybacked secondary signal does not affect the primary system.

II. SYSTEM MODEL AND PROPOSED SCHEME

A. Outline

Fig.1 shows the system model proposed in this manuscript. A pair of a transmitter and a receiver, Tx1 and Rx1, is of the primary system. On the same frequency of the primary system, we introduce the secondary transmitter Tx2 and receiver Rx2.

B. Transmitter of the primary system: Tx1

The primary system uses $M$-ary PSK modulation. Thus equivalent low-pass expression of the transmitter output can be denoted as

$$x_1(t) = \sqrt{2P_1} \sum_{k=-\infty}^{\infty} g(t - kT)X_1[k], \quad (1)$$

where $T$ is a symbol duration, $P_1$ is a power of $x_1(t)$, $X_1[k]$ is the $k$th symbol defined as

$$X_1[k] \in \left\{ e^{j\frac{2\pi m}{M}} \right\} \quad m = 0, 1, \ldots, M - 1,$$

and $g(t)$ is a real-valued pulse waveform,

$$g(t) = \begin{cases} 1 & 0 \leq t < T \\ 0 & \text{otherwise}. \end{cases} \quad (3)$$
C. Transmitter of the secondary system: Tx2

As in Fig.1, the proposed secondary transmitter has a repeater (RPT) for the primary signal, and a modulator (MOD) for its own data. The modulation parameters of the primary signal, such as a carrier frequency, modulation scheme, and symbol rate, are assumed to be known at RPT and MOD of Tx2 by cognitive radio technologies.

At RPT, the received signal \( y_0(t) \) is amplified with gain \( \alpha \) and retransmitted. The output of RPT signal is expressed as

\[
x_1'(t) = \alpha y_0(t) = \alpha c_{10}(t) * x_1(t) + \alpha z_0(t),
\]

where \( z_0(t) \) is the noise component at the input of RPT and \( c_{10}(t) \) is impulse response of the channel between Tx1 and RPT. In this manuscript, for simplicity, stationary flat fading is assumed for all radio channels and thus the channel responses become as

\[
c_{ij}(t) = H_{ij}(t - \tau_{ij}),
\]

where \( |H_{ij}| \) and \( \arg(H_{ij}) \) are the propagation loss and carrier phase shift, while \( \tau_{ij} \) is the propagation delay.

At the same time, MOD modulates and transmits data of the secondary system as \( x_2(t) \), which uses \( N \)-ary PSK modulation. For simplicity, we assume the same symbol duration \( T \) and pulse shape \( g(t) \) for both primary and secondary systems. The output of MOD is synchronized to the output of RPT. Thus the output of MOD becomes

\[
x_2(t) = \sqrt{2P_2} \sum_{l=-\infty}^{\infty} g(t - lT - \tau_{10})X_2[l],
\]

where \( P_2 \) is a power of \( x_2(t) \), \( X_2[l] \) is the \( l \)th symbol defined as

\[
X_2[l] \in \left\{ e^{j2\pi n/N} \bigg| n = 0, 1, \ldots, N - 1 \right\}.
\]

As the result, the output of Tx2 becomes

\[
x_1'(t) + x_2(t) = \alpha c_{10}(t) * x_1(t) + \alpha z_0(t) + x_2(t).
\]

D. Receiver of the primary system: Rx1

The receiver of the primary system, Rx1, receives the signal from Tx1 together with the signal from Tx2. Let us denote the channel response from Tx1 to Rx1 and Tx2 to Rx1 as \( c_{11}(t) \) and \( c_{21}(t) \), respectively. Like \( c_{10}(t) \), they are assumed to be stationary and flat. Thus the input of Rx1 is expressed as follows:

\[
y_1(t) = c_{11}(t) * x_1(t) + c_{21}(t) * [x_1'(t) + x_2(t)] + z_1(t)
\]

\[
= (c_{11}(t) + c_{11}'(t)) * x_1(t) + c_{21}(t) * x_2(t) + \alpha c_{21}(t) * z_0(t) + z_1(t),
\]

where \( c_{11}'(t) = \alpha c_{10}(t) * c_{21}(t) \) is the response between Tx1 and Rx1 through RPT, and \( z_1(t) \) is additive white Gaussian noise (AWGN) at Rx1.

Since Rx1 is designed for the primary signal without considering the presence of the secondary signal, a conventional correlation receiver is used in this manuscript. Thus Rx1 demodulates the signal and yields the decision variable for the \( k \)th symbol as follows:

\[
Y_1[k] = \int_{-\infty}^{\infty} y_1(t)g(t - kT - \tau_1)e^{-j\phi_1}dt
\]

\[
= \int_{\tau_1 + kT}^{\tau_1 + (k + 1)T} y_1(t)e^{-j\phi_1}dt.
\]

In this equation, \( \tau_1 \) and \( \phi_1 \) are estimated propagation delay and phase shift discussed in Section III.

E. Receiver for the secondary system: Rx2

In contrast to the receiver of the primary system Rx1, the secondary system receiver Rx2 can be designed taking into consideration the presence of the signal of the other (primary) system.

Fig. 2 shows the block diagram of Rx2. The input of Rx2, i.e. the sum of the signals from Tx1 and Tx2 together with AWGN \( z_2(t) \), is denoted as

\[
y_2(t) = c_{12}(t) * x_1(t) + c_{22}(t) * [x_1'(t) + x_2(t)] + z_2(t)
\]

\[
= (c_{12}(t) + c_{12}'(t)) * x_1(t) + c_{22}(t) * x_2(t) + \alpha c_{22}(t) * z_0(t) + z_2(t),
\]

where \( c_{12}(t) \) and \( c_{22}(t) \) are channel responses from Tx1 and Tx2 to Rx2, which are assumed to be stationary and flat. As in (9), \( c_{12}'(t) = \alpha c_{10}(t) * c_{22}(t) \) is the response between Tx1 and Rx2 through RPT.

Rx2 first demodulates the received signal as Rx1 does, and it yields the decision variable for the \( k \)th symbol timing of the signal of the primary system as follows:

\[
Y_2[k] = \int_{-\infty}^{\infty} y_2(t)g(t - kT - \tau_2)e^{-j\phi_2}dt
\]

\[
= \int_{\tau_2 + kT}^{\tau_2 + (k + 1)T} y_2(t)e^{-j\phi_2}dt,
\]

where \( \tau_2 \) and \( \phi_2 \) are delay and phase shift corresponding to the \( k \)th symbol of the primary system. In this manuscript, it is assumed that relayed primary signal component \( c_{22}(t) * x_1'(t) \) has larger power than \( c_{12}(t) * x_1(t) \) component. Thus \( \tau_2 \) and \( \phi_2 \) synchronized to \( \tau_1 + \tau_{22} \) and \( \phi_1 + \phi_{22} \) as discussed in Section III.

Based on the decision variable \( Y_2[k] \), Detector1 recovers the transmitted symbol of the primary system as \( \hat{X}_1[k] \). Thus the symbol is re-modulated, passed to the channel response \( c_{12}(t) + c_{12}'(t) \), and the demodulated again to yield \( Q[k] \), which is the estimate of the primary signal component in \( Y_2[k] \). Subtracting \( Q[k] \) from \( Y_2[k] \), we have secondary signal component with noise and residual primary signal component as

\[
R_2[l] = Y_2[k] - Q[k].
\]
III. SYNCHRONIZATION AT RECEIVERS

As described in subsection II-D, Rx1 demodulates symbols based on the estimated delay and phase shift, \(\tau_1\) and \(\phi_1\). For the estimation, it is assumed that Tx1 transmits a known training sequence. Then Rx1 finds a pair of \((\tau_1, \phi_1)\) that makes the correlation of sequence of demodulated decision variables and the training sequence maximum. Thus the estimate of the pair of \((\tau_1, \phi_1)\), that is \((\hat{\tau}_1, \hat{\phi}_1)\), becomes as follows:

\[
(\hat{\tau}_1, \hat{\phi}_1) = \operatorname{arg\,max}_{\tau_1, \phi_1} \mathbb{E}[\Re\{Y_1[k]X_1'[k]\}],
\]

(14)

where \(\mathbb{E}[\cdot]\) represents ensemble average, which is realized by exchanging it to time average of a certain length of the training sequence.

Combining (9) and (10), we have \(Y_1[k]\) as a sum of primary signal components (direct and relayed), secondary component, and noise components (of RPT and Rx1). Since the secondary signal and noise components are zero-mean and independent to the transmitted data of the primary system, only the sum of the primary signal components in \(Y_1[k]\) contribute the correlation in (14). This term is expressed as

\[
W_1[k] = \int_{\tau_1+kT}^{\tau_1+(k+1)T} H_{11} x_1(t-\tau_1)dt + \int_{\tau_1+kT}^{\tau_1+(k+1)T} H_{11}' x_1(t-\tau_1')dt,
\]

(15)

where

\[
H_{11}' = |\alpha H_{10} H_{21}| \exp\{j(\alpha(\phi_1) + \arg(H_{10}) + \arg(H_{21}))\},
\]

and \(\tau_1' = \tau_1 + \tau_2\). Thus \(c_1'(t) = H_1'[\delta(t - \tau_1')]\).

Since \(W_1[k] = \mathbb{E}[Y_1[k]]\) for a given \(X_1[k]\), (14) can be modified as

\[
(\hat{\tau}_1, \hat{\phi}_1) = \operatorname{arg\,max}_{\tau_1, \phi_1} \Re\{W_1[k]X_1'[k]\}.
\]

(17)

Solving this equation we have the following estimates:

\[
(\hat{\tau}_1, \hat{\phi}_1) = \begin{cases} 
(\tau_1, \phi_1) & \text{for } |\tau_1' - \tau_1| \geq T \quad \text{and } \gamma_{h1} < 1 \\
(\tau_1', \phi_1') & \text{for } |\tau_1' - \tau_1| < T \quad \text{and } \gamma_{h1} > 1 \\
(\tau_1', \phi_1) & \text{for } |\tau_1' - \tau_1| < T \quad \text{and } \gamma_{h1} < 1 \\
(\tau_1, \phi_1') & \text{for } |\tau_1' - \tau_1| \geq T \quad \text{and } \gamma_{h1} > 1,
\end{cases}
\]

where

\[
\gamma_{h1} = \frac{|H_{11}'|^2}{|H_{11}|^2}
\]

(18)

is the power ratio of the relayed and direct components of the primary signal. The phase shift estimates \(\delta_1\) and \(\delta_1'\) are given as follows:

\[
\delta_1 = \tan^{-1} \frac{T \sin(\phi_1) + (T + \tau_1' - \tau_1) \sqrt{\gamma_{h1} \sin(\phi_1')}}{T \cos(\phi_1) + (T + \tau_1' - \tau_1) \sqrt{\gamma_{h1} \cos(\phi_1')}}
\]

\[
\delta_1' = \tan^{-1} \frac{T + \tau_1' - \tau_1}{T \sin(\phi_1) + (T + \tau_1' - \tau_1) \sqrt{\gamma_{h1} \cos(\phi_1')}} + T \gamma_{h1} \cos(\phi_1')
\]

(20)

The derivation of (18) and (20) is omitted because of the page limit.

The values used in Rx2, \(\tau_2\) and \(\phi_2\), can be calculated in the same way.

IV. ANALYSIS OF SYMBOL ERROR RATE

A. Symbol Error Rate at Rx1

If \(\gamma_{h1} > 1\), and \(\tau_{11}' - \tau_{11} = n_0 T + \tau_0\) where \(n_0\) is an integer and \(0 \leq \tau_0 < T\), the decision variable \(Y_1[k]\) is expressed as

\[
Y_1[k] = \begin{cases} 
\sqrt{P_1} H_{11} X_1[k + n_0] & \text{for } n_0 < \frac{T}{2} \\
\sqrt{P_1} H_{11} X_1[k + n_0 + 1] & \text{for } n_0 + 1 \leq \frac{T}{2} + \frac{T}{2}
\end{cases}
\]

(21)

where \(\gamma_{h1}[k]\) is the sum of terms corresponding information symbols.

As \(z(t)\) and \(z_1(t)\) are independent AWGN, \(Z_0[k]\) and \(Z_1[k]\) are independent zero-mean complex Gaussian random variables. Let us define the variance of them as \(\sigma_0^2\) and \(\sigma_1^2\). Since only \(Z_0[k]\) and \(Z_1[k]\) are random variables in (21), \(Y_1[k]\) is a complex Gaussian random variable with mean \(\gamma_{h1}[k]\) and variance \(|H_{21}|^2 \sigma_0^2 + \sigma_1^2\).

When \(\gamma_{h1} > 1\) Rx1 synchronize to the relayed signal, so the region of a correct decision by \(Y_1[k]\) can be defined as

\[
D_1 = \{ |\arg(Y_1[k]) - \arg(X_1[k])| < \pi/M \},
\]

(22)

and probability of a correct decision for given interfering symbols becomes as

\[
P_{C1} \left( \hat{X}_1[k] = X_1[k] \mid X_2[l], X_1[n_0], X_1[n_0 + 1] \right) = \int_{D_1} p_1(Y_1[k])dY_1[k],
\]

(23)

where \(p_1(Y_1[k])\) is a probability density function of the complex value \(Y_1[k]\). Using this, we have the symbol error rate \(P_{M1}\) as

\[
P_{M1} = 1 - \mathbb{E} \left[ P_{C1} \left( \hat{X}_1[k] = X_1[k] \mid X_2[l], X_1[n_0], X_1[n_0 + 1] \right) \right],
\]

(24)

where \(\mathbb{E}[\cdot]\) is ensemble average for all possible values of \(X_2[l], X_1[n_0]\), and \(X_1[n_0 + 1]\).
B. Symbol Error Rate at Rx2

If $\gamma_{h2} > 1$, and $\tau'_{12} - \tau_{12} = n_0T + \tau'_0$ where $n_0$ is an integer and $0 \leq \tau'_0 < T$, the decision variable $Y_2[k]$ can be expressed like $Y_1[k]$ as

$$Y_2[k] = \left( T - \tau'_0 \right) \sqrt{P_1} H_{12} X_1[k + n'_0] + \sqrt{P_1} H_{12} X_1[k + n'_0 + 1] + \sqrt{P_2} T H_{12} X_2[l] e^{-j\phi_2}$$

$$+ \alpha H_{22} Z_0[k] + Z_2[k]$$

where $Y_2[k]$ is the sum of terms corresponding information symbols. Let us define the variance of $Z_2[k]$ as $\sigma^2_2$. Then $Y_2[k]$ is a complex Gaussian random variable with mean $\mathbf{Y}$ and variance $|H_{22}|^2 \sigma^2_2$.

As Rx1, Detector 1 outputs an estimates on $X_1[k]$, denoted as $\hat{X}_1[k]$. Let us define a region

$$D'_1 = \{ \arg(Y_2[k]) - \arg(X_1[k]) < \pi/M \}$$

then the probability that $\hat{X}_1[k] = X_1[k]$, where $X_1[k]$ has set of (2), becomes as

$$P(X_1[k] = X_1[k]) = \int_{D'_1} p'_1(Y_2[k])dY_2[k]$$

$$= \int_{D'_1} p'_1(Y_2[k])dY_2[k]$$

where $p'_1(Y_2[k])$ is a probability density function of the complex Gaussian value $Y_2[k]$.

In the Rx2, next operation is to regenerate the primary signal components in $Y_2[k]$ for interference cancellation. For this purpose the output of Detection 1, $\hat{X}_1[k]$ is passed to the channel $e_{12}(t) + \epsilon'_{12}(t)$ and then re-modulated. As the result, we have regenerated signal as follows:

$$Q[k] = (1 - \tau_0') H_{12} \hat{X}_1[k + n_0'] + \tau_0' H_{12} \hat{X}_1[k + n_0' + 1] + H'_{12} \hat{X}_1[k] e^{-j\phi_2}$$

(28)

The result of the cancellation is thus

$$R_2[l] = Y_2[k] - Q[k]$$

$$= Y_2[k] - Q[k] + \alpha H_{22} Z_0[k] + Z_2[k].$$

(29)

Note that $R_2[l]$ is a complex Gaussian random variable with mean $\mathbf{Y}$ and variance $|H_{22}|^2 \sigma^2_2$. The region of a correct decision of $X_2[l]$ by $Y_2[l]$ then can be defined as

$$D_2 = \{ |\arg(R_2[l]) - \arg(X_2[l])| < \pi/N \}$$

(30)

and probability of correct decision for given interfering symbols becomes as

$$P_{C2}(\hat{X}_2[l] = X_2[l]|X_1[n_0'], X_1[n_0' + 1]) = \int_{D_2} p_2(R_2[l])dR_2[l].$$

(31)

where $p_2(R_2[l])$ is a probability density function of the complex value $R_2[l]$. Finally, the symbol error rate $P_{M2}$ is

$$P_{M2} = 1 - E \left[ P_{C2}(\hat{X}_2[l] = X_2[l]|\hat{X}_1[k], X_1[n_0'], X_1[n_0' + 1]) \right]$$

(32)

where $E[\cdot]$ is ensemble average for all possible equal probable $X_1[n_0'], X_1[n_0' + 1]$, and $X_1[k]$ whose distribution is defined by (27).

V. NUMERICAL RESULT

This section shows the performance of the proposal scheme using QPSK modulation both primary and secondary systems ($M = N = 4$). For simplicity, no noise at RPT input ($\gamma_{n}(t) = 0$), and the same noise level at the input of Rx1 and Rx2 are assumed. In addition, the power ratio of the secondary and the relayed primary signals at Tx2 is defined as

$$\lambda = 10 \log \frac{P_2}{|H_{21}|^2 \sigma^2_2}.$$

(33)

A. Symbol error rate of Rx1

Fig.3 is the symbol error rate (SER) at Rx1, where $\tau'_{11} - \tau_{11} = 0.5T$, and $\lambda = -5$[dB]. This figure shows that the performance of the primary system is not degraded and even improved by the presence of the secondary system. In fact, the signal bearing data of the secondary system, $x_2(t)$, may degrade the performance of Rx1 as interference. But at the same time the relayed primary signal $x'_1(t)$ supports the quality of Rx1. As the result of the trade-off of these two effects, Tx2 improves performance of Rx1.

Fig.4 shows the influence of the difference of symbol timing of direct and relayed primary signal at Rx1 on SER performance. It is found that the time difference $\tau'_{11} - \tau_{11}$ affect performance much. If the difference is less than a symbol duration $T$, large difference results in worse performance. When the difference is more than a symbol duration, the performance of Rx1 is almost the same for any $\tau'_{11} - \tau_{11}$.

It is noteworthy that the presence of the secondary system always improves SER at Rx1 if the relayed primary signal is enough large at Rx1, say $\gamma_{h1} = 15$[dB].

B. Symbol Error Rate of Rx2

In Figs.3 and 4, we have confirmed that the primary system enjoys the good quality of communication if $\gamma_{h1}$ is large and $\lambda$ is small. The next question is the performance of the secondary system under the same condition.
Fig. 4. The symbol error rate at Rx1 based on $\tau_{11}' - \tau_{11}$ with $\gamma_{11} = 10, 15$[dB]

Fig.5 shows the SER performance of the secondary system with the same phase and time differences as in Fig.3. Note that if $|H_{22}|/|H_{12}| \approx |H_{21}|/|H_{11}|$, then $\gamma_{21} \approx \gamma_{11}$. In this situation, the curve of $\lambda = -5$[dB] in Fig.5 shows the SER of the secondary system when $\gamma_{11} = 20$[dB] in Fig.3. Though the SER of the secondary system under the presence of the primary signal is worse than that without the primary signal, still the degradation is less than $5$[dB]. Hence, if the secondary system accepts this degradation, it can communicate at the same spectrum with the primary system without harm to it.

It is also interesting to note that there is an optimum $\lambda$, which makes SER smallest. This is the result of the trade-off between the interference itself and accuracy in interference cancellation in Rx2. In the proposed scheme, the secondary system can transmit its signal without harm to the primary system, but with the cost in SNR.

Fig.6 shows the condition where the secondary system does not degrade the primary system and the necessary cost in SNR that compensates the SER of the secondary system. In this figure, $\gamma_{h1}$ and $\gamma_{h2}$ are set to be the same value $\gamma_{h}$. The solid line shows the upper bound of $\lambda$ for the primary system. For a given value of $\gamma_{h1}$ and $\gamma_{h2}$, if $\lambda$ below this curve is taken, then the secondary system does not degrade (even does improve) the SER of the primary system.

The dotted lines represent the condition that the secondary system keeps its SER. For example if a combination of $\lambda$ and $\gamma_{h}$ is selected from the hatched area, then the secondary system can maintain the SER of no interference by adding less than $5$[dB] to its SNR.

VI. CONCLUSION

This manuscript proposes a cognitive radio secondary system that cooperates the primary system one-way. The secondary transmitter relays the primary signal and piggybacks its own data on it.

As the result, it is found that the proposed secondary system can communicate on the same frequency of the primary system at high data rate without harm to the primary system. In addition, the condition where the primary system has no degradation by the secondary system while the secondary system can maintain its SER is clarified.

The results given in this study are drawn under AWGN channel. The consideration of multi-path and non-stationary fading environments is one of future topics. In addition, the authors are now concentrating on more sophisticated secondary transmitters that employ information of the primary signal for better performance in the secondary receiver.

REFERENCES