Acquisition of a DS/SS Signal with Macro/Micro Antenna Diversity under Rayleigh Fading and Log-normal Shadowing

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Abstract—In this paper, we propose the introduction of space diversity techniques to the code acquisition of an indoor packet radio communication system with direct-sequence spread-spectrum technique. In the proposed scheme, the base station has several radio ports each with a cluster of antennas, and the mobile station also has multiple antennas. As the result, we show great performance improvements by the proposed scheme under slow and flat Rayleigh fading and log-normal shadowing environment.

I. INTRODUCTION

In indoor environment, the radio waves reflected by walls and furniture are relatively strong and they may cause multi-path fading. Since the delay spread of indoor environment is often very small, the fading tend to be flat which causes the drop of total received power.

In addition to the fading, if the sight between the transmitter and the receiver is interrupted, the signal may encounter strong attenuation called shadowing. These multi-path fading and shadowing are known to be the major factor of performance degradation of indoor radio systems [1][2].

To reduce the effect of fading and shadowing, we often use the space diversity techniques categorized into microscopic and macroscopic. Microscopic antenna diversity, in which several independent faded signal are received at a cluster of antennas of a radio-port, is known to be effective for fading[3][4]. Macroscopic antenna diversity is used to reduce the effect of shadowing using several radio-ports which is separated from others by enough distance to make the shadowing statistics independent from others[5].

In indoor DS/SS systems where the fading environment is severe, it is difficult to achieve the synchronization of pseudo-noise (PN) code. However, the introduction of space diversity techniques have been discussed mainly for the improvement of error performance after the establishment of the synchronization but not for the code acquisition itself.

The employment of space diversity technique at the receiver for the acquisition of a DS/SS signal is discussed in [6][7]. In [8] and [9] the transmit antenna diversity is introduced to the code acquisition of a DS/SS signal. These studies considers, however, only the microscopic antenna diversity for multi-path fading.

In this paper, we use macroscopic antenna diversity to the code acquisition of a DS/SS signal in addition to microscopic antenna diversity. We assume a indoor packet radio communication system, and the performance is evaluated from the viewpoints of average time of acquisition, probability of success or failure of acquisition, and necessary preamble length.

II. SYSTEM MODEL

Figure 1 illustrates the system model of the indoor wireless packet communication system discussed in this paper. We consider the uplink communication from the mobile station to the base station. A mobile station has \( M \) transmitting antenna elements and the base station has \( J \) macro-diversity branches with a cluster of \( K \) antennas.

The channel is modeled as the product of Rayleigh fading and log-normal shadowing components [1], which are mutually independent. It is assumed that transmitter antennas and also the receiver antennas in a cluster are spaced each other by several wavelengths of the carrier. Thus, each pair of a transmitter antennas and receiver antennas has statistically independent fading but they are influenced by the same shadowing. We also assume that an antenna cluster of each macro-diversity branch is separated from others by enough distance to make the statistic of shadowing effect independent from others.

At the transmitter, the same data is transmitted from the \( m \)-th transmitter antenna is then given by

\[
s_m(t) = \sqrt{2S/M} c_m(t) \cos \omega_0 t,
\]

where \( S \) is the total transmit power, \( c_m(\cdot) \) is the PN code assigned for the \( m \)-th transmitter antenna, and \( \omega_0 \) is the angular carrier frequency. The transmit power at each transmitter antenna is normalized by \( 1/M \) to keep the total transmit power unity.

The receiver proposed in this paper is shown in Fig.2.
identically distributed (i.i.d.) Rayleigh random variable with the pdf
\[ P_{\beta_{mjk}}(x) = 2x \exp\left(-x^2\right) \quad x \geq 0 \quad \text{for all} \quad m, j, k, \]
(5)
where the mean square value of $\beta_{mjk}$ is assumed to be unity.

III. ACQUISITION CIRCUIT

In the proposed scheme, the acquisition is performed using selective macro-diversity and combining micro-diversity. In macro-diversity, we have to select the branch with the largest received power. In DS/SS systems, however, we cannot have accurate measure of the received power until the synchronization of PN code is established. So, in this paper, we first performed acquisition of PN code using the samples from $J$ macro-diversity branches, and after this process, we select one radio-port.

A. Decision Variables for Code Acquisition

The proposed acquisition circuit is shown in Fig.3, and the IQ-MF in this figure is shown in Fig.4. In Fig.4, $M_f$ is the matched filter matched with the $m$-th PN code. Since, we consider a packet radio communication system, the acquisition process is expected to be completed during the preamble of a packet which has no data modulation.

In the acquisition circuit (Fig.3), the output of each antenna is fed to a bank of IQ-matched filters (Fig.4) corresponding to $M$ PN codes of the transmitter. The output of the $m$-th IQ-MF for the $k$-th antenna in $j$-th macro-diversity branch (Fig.3(a)) is represented as
\[ R_{mjk}(t) \]
\[ = \left( \sqrt{SZ_j/M} \beta_{mjk} \Lambda_m(t - \zeta_j T_c) \cos \theta_{mjk} + n_{jk}^I(t) \right)^2 + \left( \sqrt{SZ_j/M} \beta_{mjk} \Lambda_m(t - \zeta_j T_c) \sin \theta_{mjk} + n_{jk}^Q(t) \right)^2 \]
where $n_{jk}^I(t)$ and $n_{jk}^Q(t)$ is the noise components at $I$ and $Q$ branches of IQ-MF respectively, and $\Lambda_m(t)$ is the auto-correlation function of $m$-th PN code. For simplicity, we approximate that the auto-correlation function is a simple triangle function given by
\[ \Lambda_m(t) = \int_{0}^{T_c} c(\delta) \delta d\delta \]
\[ = \left\{ \begin{array}{ll} 0 & \text{for all } m, \end{array} \right. \]
\[ \text{if } |\tau| \leq T_c, \quad T_c < |\tau| \text{for all } m, \quad (7) \]
and that the cross correlation between the different PN codes are zero. In this paper, we use the term “sync-timing”of $j$-th macro-diversity branch to refer the timing that $t = \zeta_j T_c$, which is common for all $m, k$.

The output of each IQ-MF in every branches is then summed up and we have
\[ R_j(t) = \sum_{m=1}^{M} \frac{\sum_{k=1}^{K} R_{mjk}(t)}{K} \]
(8)
at Fig.3(b). These signals are sampled to be fed to the code acquisition logic (Fig.3(c)). For simplicity, we assume that the sampling is made at the center of each chip.
B. Acquisition Process

The acquisition logic uses these samples to find the sync-timing and choose the best branch with the following algorithm, which consists of search and verification modes.

1) The search mode employs the parallel search strategy. The samples of J branches for $LT_c$ seconds, thus $JL$ in total, are stored in a memory.

2) The largest one among the JL samples is selected and considered to be corresponding to sync-timing of a branch tentatively, then the acquisition system is turned to the verification mode to test this hypothesis.

3) In the verification mode, the above hypothesis is examined with the received signals at the branch for $ALT_c$ second. If the sample is verified to be of sync-timing, the acquisition is declared completed. The system goes back to 1). In this paper, we assume that the verification mode works ideally [10][11].

C. Probability distribution of the samples

The probability density function (pdf) of the IQ-MF output at the sampling timings $R_{njk}(nT_c)$ is given by (9) and (10)[12], where $H_1$ represents the case that the sampling timing corresponds to the sync-timing, and $H_0$ is the case that the sampling timing does not correspond to the sync-timing.

$$P_{R_{njk}}(x|H_1) = f_{NC_{MK}}(x, \sigma_n^2, a_{mjk}^2, 1)$$

$$P_{R_{njk}}(x|H_0) = f_{C_{MK}}(x, \sigma_n^2, 1),$$

where

\[ a_{mjk}^2 = \frac{\beta_{mjk}^2 L^2 T_c^2 S Z_j}{M} \]

\[ \sigma_n^2 = \frac{N_0 LT_c}{2}. \]

In the above equations, $f_{NC_{MK}}(x, \cdot, \cdot, \cdot, n)$ and $f_{C_{MK}}(x, \cdot, \cdot, \cdot, n)$ are the pdf of noncentral and central chi-square distribution with $2n$ degrees of freedom, respectively [13].

Next, let us consider the pdf of the samples used in the acquisition logic, i.e., $R_q(nT_c)$. Since the samples at Fig. 5(c) is the sum of $MK$ i.i.d. chi-square random variables, each with 2 degrees of freedom, the samples $R_q(nT_c)$ follow chi-square distribution with $2MK$ degrees of freedom as follows.

$$P_{R_q}(x|H_1) = f_{NC_{MK}}(x, \sigma_n^2, a^2_1, MK)$$

$$P_{R_q}(x|H_0) = f_{C_{MK}}(x, \sigma_n^2, MK).$$

The parameter $a^2_1$ is

\[ a^2_1 = \sum_{m=1}^{M} \sum_{k=1}^{K} \frac{\beta_{mjk}^2 L^2 T_c^2 S Z_j}{M} \]

\[ = \frac{\alpha_j L^2 T^2_c S Z_j}{M}, \]

where the value $\alpha_j$ is the sum of $a^2_{mjk}$ all over $m$ and $k$, that is, $\alpha_j = \sum_{m=1}^{M} \sum_{k=1}^{K} a^2_{mjk}$. Since each $\beta_{mjk}$ is Rayleigh distributed random variable, the value $\alpha_j$ becomes central chi-square random variables with $2MK$ degrees of freedom, and has probability density function

$$P_{\alpha_j}(x) = f_{\chi^2}(x, 1/2, MK) \text{ for all } j. \quad (15)$$

IV. PERFORMANCE ANALYSIS

A. Measures of Performance

In this paper, we evaluate the performance of the proposed scheme by the following three measures.

1) Mean Acquisition Time

Mean acquisition time has been most widely used as the measure of performance of acquisition schemes. This is the expectation of the time needed to acquire the timing of PN code when the preamble length is enough large.

2) Misacquisition Probability: $P_{acq}$

In packet radio communications, the acquisition must be completed within a preamble of a packet, or the packet will be lost. Thus, as one of performance measures, we introduce misacquisition probability which is the probability that the acquisition circuit cannot acquire the timing of PN code within a given preamble length, $N LT_c$.

3) Required Preamble Length: $N_{acq}$

From the viewpoint of the efficiency of the channel capacity, the preamble length should be as short as possible. Thus, we employ the preamble length required to acquire the timing of PN code with the probability more than $P_{acq}$ as another performance measure.

B. Mean Acquisition Time

In this subsection, we derive the mean acquisition time of the proposed scheme in a similar way to [12]. The state transition diagram is shown in Fig.5. The state “S” represents the condition that the acquisition circuit is detecting the timing corresponding to the sync-timing, and the state “Acq”, which is the sole absorbing state, represents the condition that the acquisition is completed.
In this figure, \( G_1(z) \) denotes the generating function of the correct decision through the search mode and the verification mode, and \( G_2(z) \) denotes that the candidate of the search mode is rejected in the verification mode. \( G_1(z) \) and \( G_2(z) \) are given by

\[
G_1(z) = P_D z^{-(L+1)L_T e} \tag{16}
\]

\[
G_2(z) = (1 - P_D) z^{-(L+1)L_T e} \tag{17}
\]

where \( P_D \) denotes the probability that the selected sample in the search mode corresponds to a correct sync-timing.

Using these functions, the generating function of the acquisition time is obtained as

\[
G(z) = G_1(z) + G_2(z)G_1(z) + G_2^2(z)G_1(z) + \cdots = \frac{G_1(z)}{1 - G_2(z)} \tag{18}
\]

We can see that the acquisition probability \( G(1) = 1 \) when the length of the preamble is infinite. The acquisition time is a random variable due to noise, and the average of this is given by

\[
\mathbb{E}[T_{acq}] = \frac{d}{dz} \ln G(z)|_{z=1} \tag{19}
\]

where \( \mathbb{E} \) means ensemble average over the noise. With (16)-(18), (19) becomes

\[
\mathbb{E}[T_{acq}] = \frac{1 + A}{P_D} LT_e \tag{20}
\]

The detection probability \( P_D \) is the probability that the sample corresponding to sync-timing is larger than other \( (L-1) \) samples.

1) \( J = 1 \)

When we do not use macro-diversity in the base station, the detection probability \( P_D \) is represented as

\[
P_D = \int_0^\infty P_R(x|H_1) \left[ \int_0^y P_R(x|H_0)dx \right]^{L-1} dy \tag{21}
\]

From (12)-(14), (21) becomes

\[
P_D(\alpha_1, Z_1) = \int_0^\infty f_{
CI_1}(x, \sigma_n^2, -\alpha_1 L^2 T_c^2 S Z_1, MK) \cdot \left[ \int_0^y f_{CI_2}(x, \sigma_n^2, MK)dx \right]^{L-1} dy \tag{22}
\]

In this equation, we notice the detection probability \( P_D \) is the function of \( \alpha_1 \) and \( Z_1 \) in fading and shadowing channel.

Thus, the mean acquisition time is represented as

\[
\mathbb{E}[T_{acq}] = \int_0^\infty \int_0^\infty \frac{(1 + A)LT_e}{P_D(x,y)} P_{a_1}(x) P_{z_1}(y) dx dy, \tag{23}
\]

where \( \mathbb{E}[x] \) denotes the average over the attenuation due to fading and shadowing.

2) \( J \geq 2 \)

When the macro-diversity is in use in the base station, the detection probability \( P_D \) can not be derived easily. Thus, in this case, we calculate the mean acquisition time by simulation.

C. Misacquisition Probability and Required Preamble Length

When \( J = 1 \), the misacquisition probability is represented as

\[
P_{m_{acq}} = \int P_{a_1}(x) P_{z_1}(y) \left( 1 - P_D(x,y) \right) \frac{x}{\varphi} dx dy \tag{24}
\]

and we can find the required preamble length \( N_{req} \) by increasing \( N \) until \( 1 - P_{m_{acq}} > P_{acq} \) if fulfilled.

When \( J \geq 2 \), the misacquisition probability and required preamble length is not derived easily, so we derive these performances by computer simulation.

V. NUMERICAL EXAMPLES

We calculate the performance of the proposed scheme under the following conditions:

- Length of PN code : \( L = 63 \).
- Time required in the verification mode : \( ALT_e = 4LT_e \).
- Standard deviation of shadowing attenuation in dB : \( \sigma_s = 8 \).[2]

We define the average chip energy \( E_c \) as the mean value of the total received power within one chip duration at all the macro-diversity branches, and is denoted as

\[
E_c = E[ST_z, Z_f] = ST_z \overline{Z_f}.
\]

The mean acquisition time, the misacquisition probability, and the required preamble length of the proposed scheme are shown in Fig.6, Fig.7, and Fig.8, respectively. In Fig.7, we assume that the length of the preamble is 40bits, and in Fig.8, we assume that the required acquisition probability \( P_{acq} \) is 99.9%.

From these figure, it can be seen that the macro-diversity and the receive antenna diversity offer a substantial performance improvement. These figures also show that the transmit antenna diversity improves the performance when the number of the receiver antenna is one and \( E_c/N_0 \) is large. This is the result of the tradeoff between the protection against multi-path fading by transmit antenna diversity and the noncoherent combining loss due to the dispersion of the transmitted power. When \( E_c/N_0 \) is large, the former factor influences larger, and the latter dominates when \( E_c/N_0 \) is small. We also notice that the transmit antenna diversity is more effective when the macro-diversity is in use. When macro-diversity is used, the selected radio-port or branch can be expected to receive relatively large signals, hence the effect of the dispersion of the transmitted power does not influence so much. If the receiver uses plural antennas, however, the transmit
antenna diversity does not improve the performance, as the fading can be mitigated to some extent even when $M = 1$ with the receive antenna diversity, and additional transmit antenna diversity only introduces the noncoherent combining loss due to the dispersion of the transmitted power.

VI. CONCLUSION

In this paper, we propose the introduction of macroscopic receiver antenna diversity technique for the acquisition of PN code of a direct-sequence spread-spectrum signal together with microscopic transmitter and receiver antenna diversity techniques for the uplink of an indoor radio system. From the numerical examples, we have shown that both the macro and micro receive antenna diversity are very effective in the acquisition of PN code of a DS/SS signal under slow and flat Rayleigh fading and log-normal shadowing environment. We have also shown that the transmit (micro) antenna diversity is also effective in some conditions.

REFERENCES